

Antenna Servo Design for Tracking Low-Earth-Orbiting Satellites

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The upcoming NASA missions will require tracking of low-orbit satellites. As a consequence, NASA antennas will be required to track satellites at higher rates than for the current deep-space missions. This paper investigates servo design issues for the 34-m beam-waveguide antennas that track low-orbit satellites. This includes upgrading the servo with a feedforward loop, monopulse controller design, and tracking error reduction either through proper choice of elevation pinion location or through application of a notch filter or adjustment of the elevation drive amplifier gain. Finally, improvement of the signal-to-noise ratio through averaging of the oversampled monopulse signal is described.

Introduction

THE National Aeronautics and Space Administration (NASA) antenna network, called the Deep Space Network (DSN), is operated by the Jet Propulsion Laboratory. It consists of several antenna types located at three sites Goldstone, CA, Canberra, Australia, and Madrid, Spain and serves as a communication tool for space exploration. Future NASA missions will include low-orbiting satellites, which will require significantly higher antenna tracking rates. Thus the servos for the new-generation 34-m beam-waveguide (BWG) antennas should be upgraded to be able to follow commands at higher rates. These upgrades are illustrated in this paper, with the new DSS-13 BWG antenna controller design.

The existing proportional and integral (PI) controllers of the antennas satisfy the requirements for deep-space X-band (8.4-GHz) tracking. For high-rate command following a simple and reliable choice is a feedforward controller, described in this paper. For tracking, a monopulse controller is a fast-rate alternative to the existing conical scanning (conscan) technique. The design and performance of a monopulse controller is discussed. It is shown that its performance is improved through either proper choice of the location of the elevation pinion or the implementation of a notch filter or the amplifier gain adjustment. Finally, the improvement of the signal-to-noise ratio (SNR) of the monopulse signal is presented. By averaging the redundant monopulse samples, the SNR improvement ranges from 7 up to 17 dB.

Feedforward Controller Design

Tracking accuracy of fast-moving objects can be improved if a PI servo is augmented with a feedforward loop, as in Fig. 1, where G_p , G_c , G_f , and G_w denote transfer functions of the antenna's rate loop, PI controller, feedforward gain, and wind disturbance, respectively; r is a command; y is output (elevation and azimuth angles); e is tracking error in azimuth and elevation; u is plant input; and w is wind disturbance.

In order to analyze the impact of the feedforward gain on the closed-loop system performance, the transfer function from the command r and wind disturbance w to the tracking error e was derived. From Fig. 1, one obtains

$$e = r - y, \quad y = G_p u + G_w w, \quad u = G_f r + G_c e \quad (1)$$

Assuming $I + G_p G_c$ nonsingular and denoting $G_0 = (I + G_p G_c)^{-1}$,

from Eq. (1) one obtains

$$e = G_0(I - G_p G_f)r - G_0 G_w w \quad (2)$$

From the above equation it follows that in the absence of wind disturbances perfect tracking ($e = 0$) is obtained for the feedforward gain G_f such that

$$G_p(\omega)G_f(\omega) = I \quad (3)$$

In the case of the DSN antennas, condition (3) is satisfied in a certain frequency range only. By inspection of the magnitudes of the plant transfer function for azimuth command in Figs. 2a,b (transfer functions for elevation command are similar), one can see that for frequencies $0 \leq \omega \leq 2\pi$ rad/s ($0 \leq f \leq 1$ Hz) the plant transfer function G_p can be approximated with an integrator $G_p \cong G_{p0} = (j\omega)^{-1}I_2$ for $0 \leq \omega \leq 2\pi$ rad/s. Thus, the feedforward differentiation

$$G_f = j\omega I_2 \quad (4)$$

will satisfy Eq. (3) in the frequency range $0 \leq \omega \leq 2\pi$ rad/s. In Fig. 2, the diagonal terms of the differentiator transfer function (4) are shown with dot-dashed lines. Their inverses (dashed lines) are equal to the plant transfer function, as in Fig. 2, for frequencies up to 1 Hz. The off-diagonal terms of G_f (transfer functions from elevation command to azimuth position and from azimuth command to elevation position) are small for frequencies up to 1 Hz.

The closed-loop transfer function (azimuth encoder to azimuth command) for a system with and without the feedforward gain is compared in Fig. 3. The figure shows that for frequencies up to 1 Hz the system with the feedforward gain has superior tracking properties when compared with the system without feedforward gain. This is confirmed by tracking with a trajectory, as that in Fig. 4. The DSS-13 antenna, with proportional gain $k_p = 0.5$ and integral gain $k_i = 1.8$ in azimuth and elevation, was investigated. Pointing errors are shown in Fig. 5, and the maximal error of 1.4 mdeg in elevation (EL) and 0.2 mdeg in cross-elevation (XEL) was observed, which exceeds the requirements. However, for this controller the high-frequency components of the command are strongly amplified, as can be observed from the transfer function plots in Fig. 3, where the resonance peaks of the system with feedforward gain are much

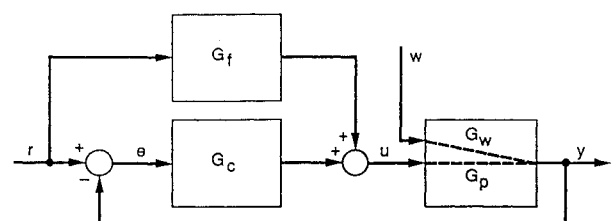


Fig. 1 PI controller with feedforward loop.

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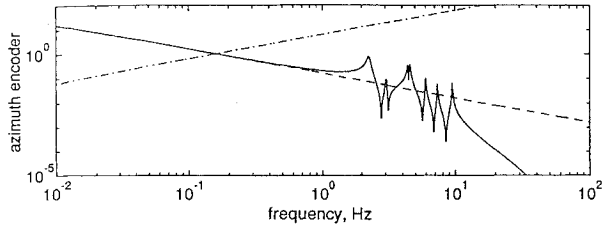


Fig. 2 Magnitudes of rate loop, differentiator, and integrator transfer functions.

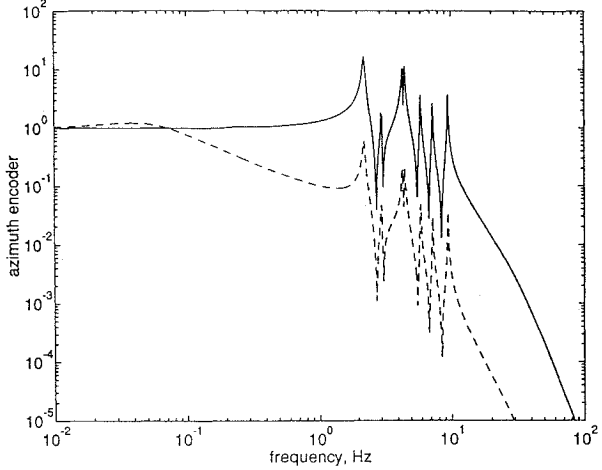


Fig. 3 Magnitudes of closed-loop transfer function: with feedforward (solid line) and without feedforward (dashed line).

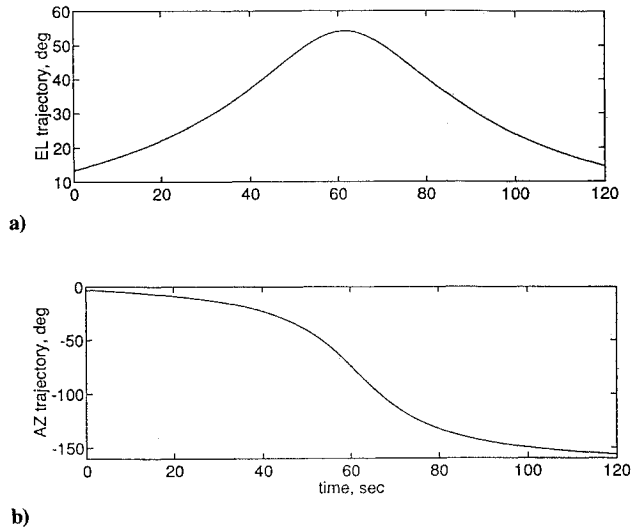


Fig. 4 Elevation and azimuth trajectories.

higher than the ones of the system without feedforward gain. As a result, any sharp change in the command may cause excessive vibrations of the antenna.

Despite the increased sensitivity to the command inputs, the disturbance rejection of the antenna with feedforward gain remains the same as that for the antenna without feedforward gain. This follows from Eq. (2), where the tracking error e due to wind disturbance w is independent of the feedforward gain G_f . Thus the pointing errors due to wind gust disturbances are comparable with the errors for the PI servo; see Ref. 1.

Monopulse Controller Design

In monopulse tracking a deviation of an antenna from a target is detected by four slightly displaced feedhorns, each of them receiving

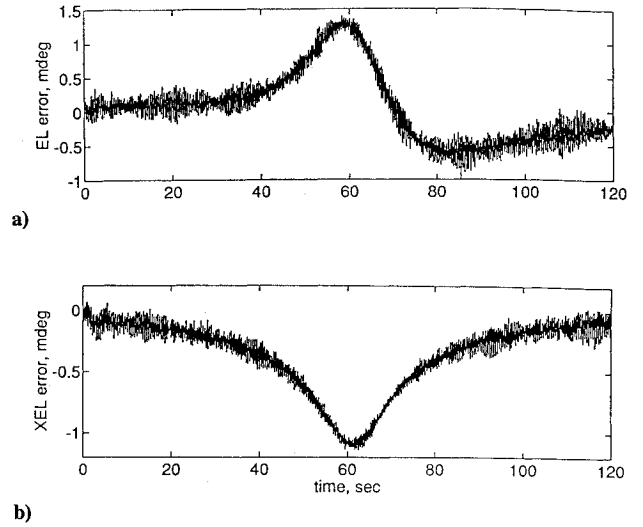


Fig. 5 Pointing errors in elevation and cross-elevation for feedforward controller.

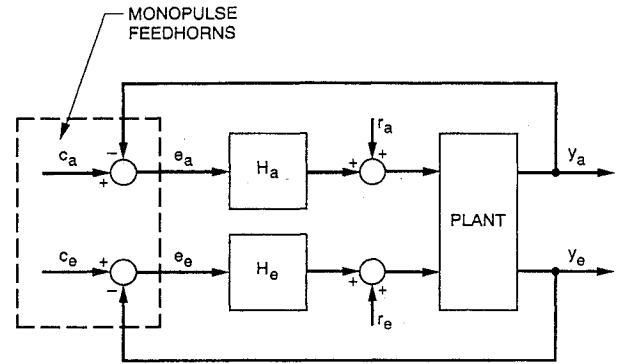


Fig. 6 Block diagram of monopulse tracking system.

the signal from a slightly different angle. The received beams are added and subtracted to form a sum and a difference beam. The difference beam is zero when the target is on the antenna boresight, and the nonzero difference beam produces an error signal, which is used by the monopulse control system. Detailed description of the monopulse technique is given in Refs. 2–5.

The monopulse tracking control system is shown in Fig. 6. It consists of the plant, monopulse feedhorns, and the monopulse controllers in azimuth (H_a) and elevation (H_e). The plant in this case is the antenna with the closed encoder position loop. The monopulse feedhorns detect the tracking errors in azimuth (e_a) and elevation (e_e). The encoder command is denoted r_a in azimuth and r_e in elevation. The feedhorns detect the tracking errors e_a and e_e directly, and the output signals y_a and y_e , as well as the commands c_a and c_e , are not available. Note that y_a and y_e signals are not the encoder outputs but antenna positions related to the focal location of the radio-frequency (RF) beam.

Denote the two-input, two-output plant transfer function G ,

$$G = \begin{bmatrix} G_{aa} & G_{ae} \\ G_{ea} & G_{ee} \end{bmatrix} \quad (5)$$

and introduce the following notations:

$$c = \begin{bmatrix} c_a \\ c_e \end{bmatrix}, \quad r = \begin{bmatrix} r_a \\ r_e \end{bmatrix}, \quad e = \begin{bmatrix} e_a \\ e_e \end{bmatrix}, \quad y = \begin{bmatrix} y_a \\ y_e \end{bmatrix}, \quad H = \text{diag}(H_a, H_e) \quad (6)$$

so that

$$y = G_c c + G_r r \quad (7)$$

where $G_c = (I + GH)^{-1}GH$ and $G_r = (I + GH)^{-1}G$ are of dimension 2×2 . The components of G have the following properties:

$$|G_{aa}| \cong |G_{ee}| \cong 1 \quad \text{for } f < f_0 \quad (8a)$$

$$|G_{aa}| \ll 1, \quad |G_{ee}| \ll 1 \quad \text{for } f \gg f_0 \quad (8b)$$

$$|G_{ae}| \ll 1, \quad |G_{ea}| \ll 1 \quad \text{for all } f \quad (8c)$$

as illustrated in Fig. 7. The above properties yield the following monopulse loop properties:

$$y \cong c \quad \text{for } \|GH\| \gg 1 \quad (9a)$$

$$y \cong r \quad \text{for } \|GH\| \ll 1 \quad (9b)$$

$$y \cong 0.5(c + r) \quad \text{for } GH \cong I \quad (9c)$$

In the first case of large open-loop gain the closed-loop monopulse system follows the monopulse command. In the second case of small open-loop gain the closed-loop system follows the encoder command. In the last case of unit monopulse gain the system follows the average of the monopulse and the encoder command. In order to prove this, note that for $\|GH\| \gg 1$, one obtains $\|G_r\| \ll 1$ and $\|G_c\| \cong I$, and hence $y \cong c$; for $\|GH\| \ll 1$ one obtains $\|G_c\| \ll 1$ and $G_r \cong G$, and thus $y \cong r$; for $GH \cong I$ one obtains $G_r \cong 0.5G$, and thus, from Eq. (7), $y = 0.5GHc + 0.5Gr$, and since $G \cong I$ [see Eq. (8)], one obtains $y = 0.5(c + r)$.

The transfer function H of the monopulse controller is determined as follows. The monopulse bandwidth f_m is smaller than the encoder bandwidth f_0 ; therefore the monopulse tracker will compensate for slowly varying error signal e . If condition (9a) is satisfied for $f < f_m$, the monopulse tracking system will follow the command c . And since $G \cong I$ for $f < f_m$, $\|H\| \gg 1$ is required to satisfy condition (9a). In addition, a rapid roll-off rate for $f > f_m$ would be an advantage. However, the roll-off rate is limited through

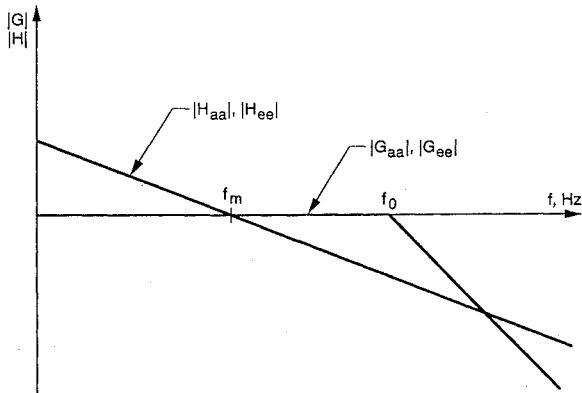


Fig. 7 Magnitudes of G and H .

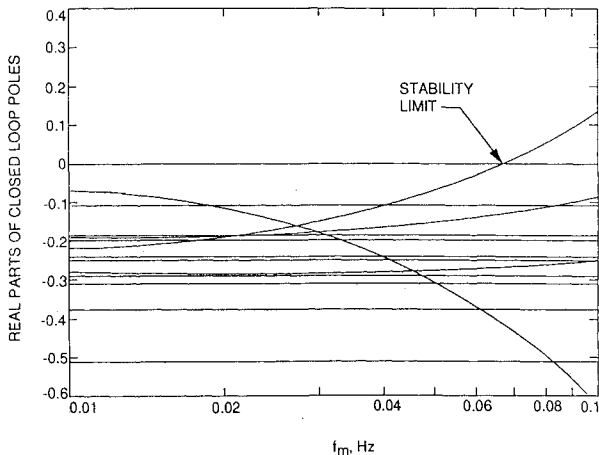


Fig. 8 Real parts of closed-loop poles vs f_m .

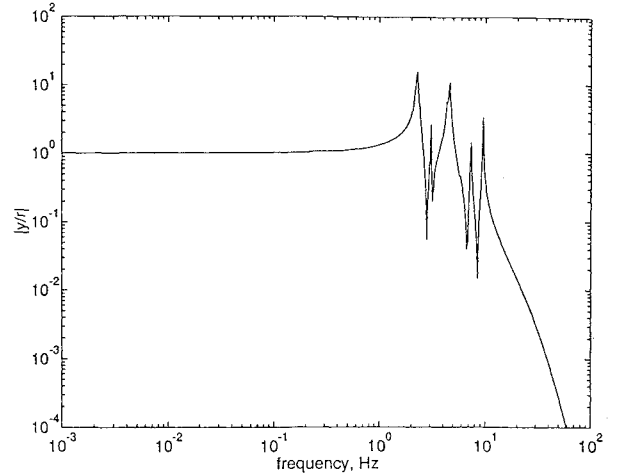


Fig. 9 Magnitude of plant transfer function from input r to output y for elevation angle command.

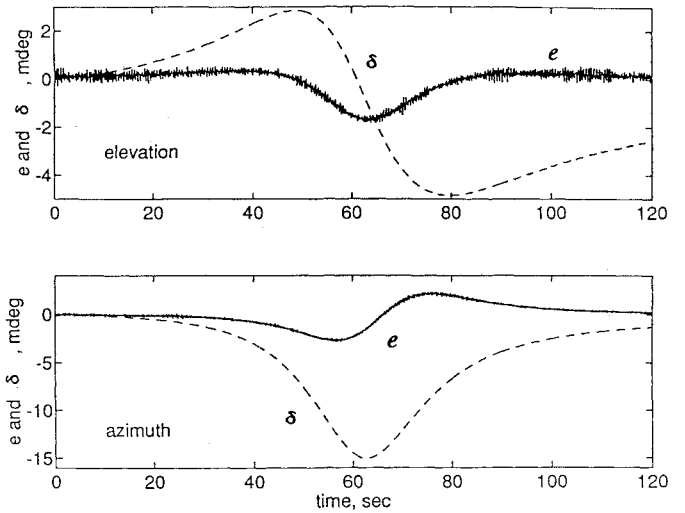


Fig. 10 Tracking error e and deviation δ : a) in elevation; and b) in azimuth.

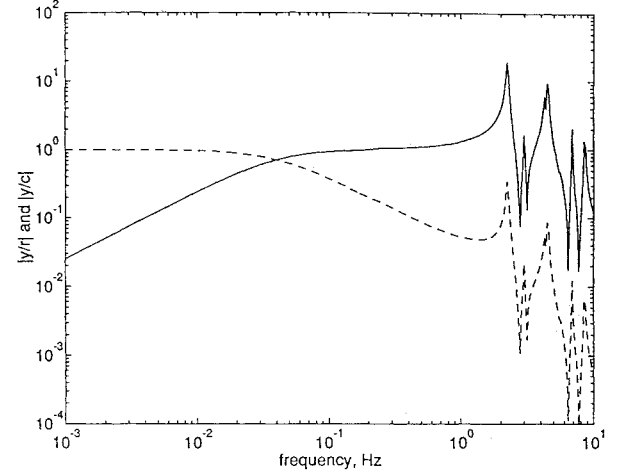


Fig. 11 Magnitudes of closed-loop transfer function (azimuth angle command) from input r to output y (solid line) and input c to output y (dashed line).

the Bode conditions, as specified in Ref. 6. Namely, the roll-off rate in the region of the gain crossover frequency must not exceed 40 dB/decade, and for reasonable stability margin it must actually be smaller than this. Hence the following transfer function of the monopulse tracker is chosen:

$$H = \frac{2\pi f_m}{s} I_2 \quad (10)$$

This transfer function satisfies Eq. (9a) for $f < f_m$ and has a roll-off rate of 20 dB/decade for $f > f_m$ (see Fig. 7). The parameter f_m of H is determined by analyzing the root locus of the monopulse closed-loop system with respect to f_m . The plot of real parts of closed-loop poles is shown in Fig. 8. It shows that for $f_m \geq 0.067$ Hz the monopulse system is unstable. In order to maintain a reasonable stability margin, $f_m = 0.04$ Hz is chosen.

The plant transfer function G is obtained for the DSS-13 antenna with the encoder loop closed and the feedforward loop implemented. The magnitudes of the plant transfer function are shown in Fig. 9 for elevation command (a similar result can be obtained for azimuth command). From the figure one can see that conditions (8a) and (8b) are satisfied, but condition (8c) is violated for some frequencies from the interval $f = [2, 10]$ Hz. This violation will cause some performance deterioration.

The azimuth and elevation components of the command signal r are shown in Fig. 4. The command c is slightly deviated from r by δ , i.e., $c = r + \delta$, where $\|\delta\| \ll \|r\|$. The plot of δ is shown in Fig. 10. Magnitudes of transfer functions are shown in Fig. 11 from input r to output y (solid line) and from input c to output y (dashed line). They indicate that the system follows low-frequency command c , high-frequency command r , and low- and high-frequency command u .

An implementation of the monopulse controller requires its discretization in time. The monopulse signal is supplied with the rate f_d hertz or the sampling time $T = 1/f_d$ seconds. In the case of the DSS-13 antenna the sampling rate is 10 Hz. The main difference between the continuous-time and the discrete-time tracker lies in a delay of the tracking error. The monopulse closed-loop systems with sampling rates 10 and 50 Hz have been simulated. The 50-Hz sampled system has been simulated for evaluation of accuracy of the slower sampled 10-Hz system. The simulations show similar results for 10- and 50-Hz sampling and are shown in Fig. 10 for the 10-Hz sampled system, where the solid line denotes the tracking error e and the dashed line the deviation δ . The plots show that the pointing accuracy increased more than twofold in both cases. A sampling rate of 10 Hz is satisfactory to maintain the accuracy of the control system, and the 0.1-s delay does not degrade the system performance.

Improving Tracking Performance

As mentioned before, the implementation of the feedforward loop causes a significant excitation of flexible motion of the antenna, specifically in the elevation loop. The mode of deformation for the highest peak in the elevation-to-elevation transfer function is shown in Fig. 12. It is a bending mode of the antenna structure, strongly excited not only by the elevation command, but also by the azimuth command. It impacts the stability and performance of an antenna. This mode is extremely difficult to control with elevation and/or azimuth torques, but one of the measures described below can be taken to reduce the impact of this mode on tracking performance:

proper location of the elevation pinion or application of a notch filter or adjustment of the amplifier gain in elevation drive.

Choosing Elevation Pinion Location

The antenna tracking errors in elevation (EL) and cross-elevation (XEL) for the three positions of the elevation pinion ($\alpha = 0$ deg, 60 deg, 90 deg), as in Fig. 12, are compared in Table 1. It shows that the higher the pinion, the smaller the error. The decrease is almost proportional to $\cos \alpha$ and can be explained by the fact that the elevation-to-elevation mode is excited mainly by the horizontal component F_h of the elevation pinion force F_t , proportional to $\cos \alpha$, cf. Fig. 12.

Implementing a Notch Filter

The critical elevation-to-elevation peak can be decreased by applying a notch filter in series with the rate-loop model. This narrow-band filter removes a component of the specified frequency from the signal. The notch filter transfer function is as follows:

$$G_f(s) = \frac{s^2 + \omega_f^2}{s^2 + 2\delta\omega_f s + \omega_f^2} \quad (11)$$

where ω_f radians per second is filter frequency and δ is a damping coefficient that defines the bandwidth of the filter. The elevation input signal to the antenna excites the antenna vibration mode of frequency $f_f = 2.12$ Hz; thus $\omega_f = 2\pi f_f = 13.32$ rad/s and $\delta = 0.2$ have been chosen. In implementation, the matching of the filter frequency and the antenna resonance frequency is not a difficult task, since this particular resonance peak is strong and dominant and thus easily detected.

The closed-loop properties of the antenna with and without a notch filter are compared in Fig. 13, where for the system with the notch filter the peak at 2.12 Hz has disappeared. The reduction of the peak allows one to increase significantly the gain of the monopulse loop without losing stability. This gain yields about a 10-fold reduction of elevation pointing error, as follows from comparison of Figs. 14 and 10a. Additional simulations performed to test the robustness of the system to filter frequency variations up to 10% of the nominal frequency show negligible deterioration of performance.

Adjusting Amplifier Gain

The impact of the critical elevation-to-elevation peak on monopulse controller stability and performance can be reduced by ad-

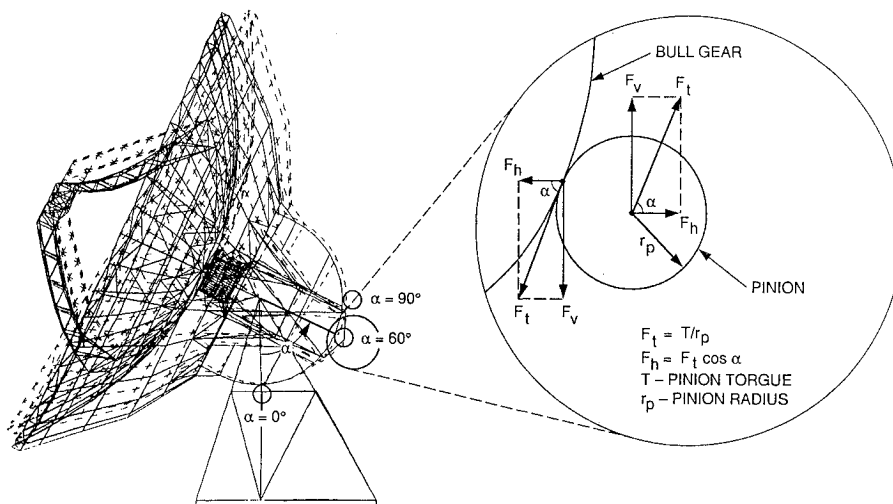


Fig. 12 Bending mode of antenna and elevation pinion locations.

Table 1 Tracking errors, mdeg

	EL	XEL	Total
Pinion at 0 deg	1.43	0.14	1.44
Pinion at 60 deg	0.76	0.08	0.77
Pinion at 90 deg	0.35	0.07	0.36

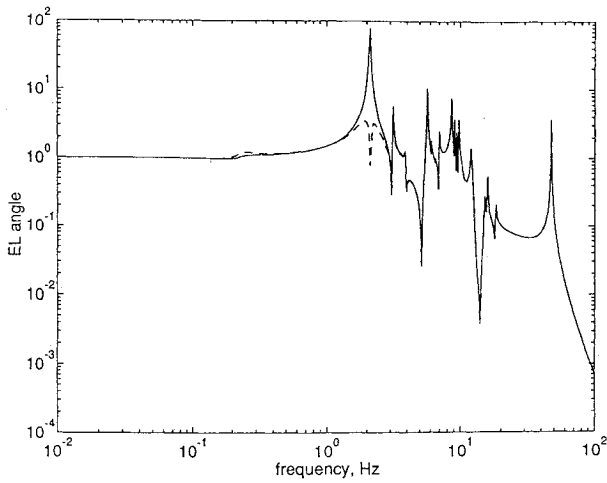


Fig. 13 Transfer functions (elevation command to elevation encoder) of closed-loop antenna: without (solid line) and with (dashed line) notch filter.

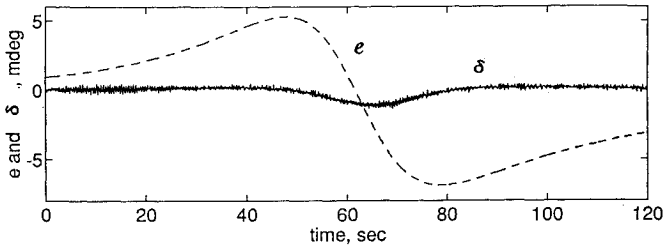


Fig. 14 Elevation pointing error for antenna with notch filter.

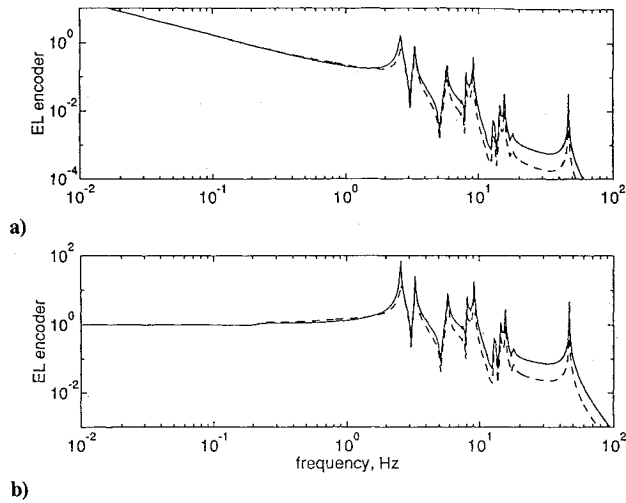


Fig. 15 Transfer functions, elevation rate command to elevation encoder, for nominal $k_r = 160$ (solid line) and $k_r = 160/3$ (dashed line): a) of rate-loop model; b) of closed-loop antenna.

justing the open-loop gain. For example, the gain can be adjusted by varying the elevation drive amplifier gain. The nominal gain k_r of the elevation drive amplifier is lowered to $k_{r0} = \beta k_r$, where $\beta = 0.33$ has been chosen through simulated tests. In doing so, the rate-loop transfer function has been lowered for high frequencies, as in Fig. 15a. But the feedback gain, which contains an integrator, retains the tracking properties for low frequencies of the closed-loop transfer function, whereas the higher frequency part of the transfer function is not compensated, lowering the critical peak; cf. Fig. 15b. This simple approach allows one to increase gain of the monopulse loop, producing an improvement in tracking performance similar to that with the notch filter in Fig. 14. The explanation is as follows: Let G be the transfer function of the rate-loop model from elevation to elevation and K the transfer function of the PI controller. In the closed-loop transfer function $G_0 = KG/(1 + KG)$, K consists of an integrator. Therefore $|K| \rightarrow \infty$ for $\omega \rightarrow 0$ and $|K| \rightarrow 0$ for

$\omega \rightarrow \infty$. It yields $|G_0| \rightarrow 1$ for $\omega \rightarrow 0$ and $|G_0| \rightarrow 0$ for $\omega \rightarrow \infty$; thus tracking for low frequencies is preserved and the peaks in higher frequencies are suppressed. This can be seen in Fig. 15b, where the low-frequency part of the closed-loop transfer function is the same, equal to 1 for $\beta = 1$ and $\beta = 0.33$, but for higher frequencies the transfer function for $\beta = 0.33$ is lower than for $\beta = 1$.

Improving SNR

A monopulse signal is typically contaminated with measurement noise of significant intensity. Noise intensity is measured with

$$\text{SNR} = 10 \log_{10} \frac{P_s}{P_n} \quad [\text{dB}] \quad (12)$$

where P_s and P_n are signal and noise powers, respectively. The noise impacts the pointing accuracy of the control system. Here, a simple method that improves SNR is discussed.

The monopulse signal $u(i\Delta t)$ consists of a true measurement $u_0(i\Delta t)$ and a noise $n(i\Delta t)$:

$$u(i\Delta t) = u_0(i\Delta t) + n(i\Delta t) \quad (13)$$

where $u_0(i\Delta t) = y_0(i\Delta t) - y(i\Delta t)$. It is assumed initially that the noise $n(i\Delta t)$ is a white noise with zero mean, $E[n(i\Delta t)] = 0$, where $E[\cdot]$ is the expectation operator. The assumption is the worst-case scenario. White noise consists of components of all frequencies of equal intensity s_0 , up to the Nyquist frequency f_c . Typically, the measurement noise is rather a high-frequency noise; thus its impact on system performance is less severe than the white noise (the closed-loop antenna bandwidth is 1–2 Hz).

The monopulse signal $u(i\Delta t)$, shown in Fig. 16 for sampling time $\Delta t = 0.02$ s, is transmitted to the antenna controller in clusters every N samples (typically $N = 5$); thus the new sampling period is $\Delta T = N\Delta t$, and a cluster $U(i\Delta T) = \{u_1(i\Delta T), u_2(i\Delta T), \dots, u_N(i\Delta T)\}$ consists of N measurements $u_k(i\Delta T)$:

$$u_k(i\Delta T) = u(i\Delta T + k\Delta t), \quad k = 1, \dots, N \quad (14)$$

The mean value $m_k = E[u_k(i\Delta T)]$ and the variance $\sigma_k = E[\Delta u_k(i\Delta T)]^2$ of each component are the same in the cluster:

$$m_k = m_N, \quad \sigma_k^2 = \sigma_N^2, \quad k = 1, \dots, N \quad (15)$$

This assumption has the following meaning: The value of $u_0(i\Delta t)$ is considered constant within the period ΔT if the reaction of the antenna to $u_0(t_0 + i\Delta t)$ is the same as to $u_0(t_0 + N\Delta t)$ for $i = 1, \dots, N$. This property has been confirmed by the earlier simulations reported in Ref. 7.

Although the monopulse signal is sent to the controller in clusters, only the last component, $u_N(i\Delta T) = u(i\Delta T + N\Delta t)$, is used to drive the monopulse controller. This excess information is used to reduce the SNR by averaging the signal within a cluster. The average value $u_{av}(i\Delta T)$ of the monopulse signal within the cluster of N samples is obtained as

$$u_{av}(i\Delta T) = \frac{1}{N} \sum_{k=1}^N u_k(i\Delta T) \quad (16)$$

It is shown in the Appendix that in the case of white noise the mean value m_{av} of the averaged process, $u_{av}(i\Delta T)$, and the mean value m_N of the nonaveraged process, $u_N(i\Delta T)$, are the same, whereas the variance of the averaged process (σ_{av}^2) is smaller than the variance

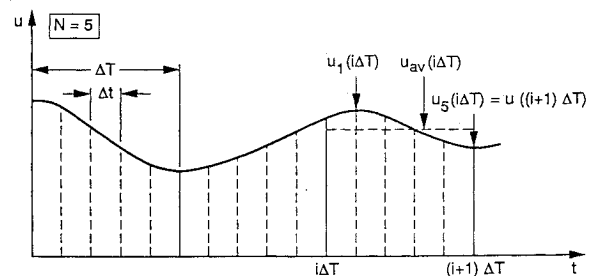


Fig. 16 Monopulse signal.

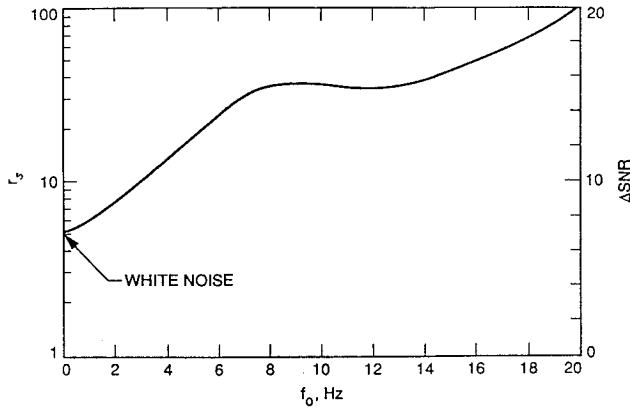


Fig. 17 Variances ratio and SNR increase vs cutoff frequency.

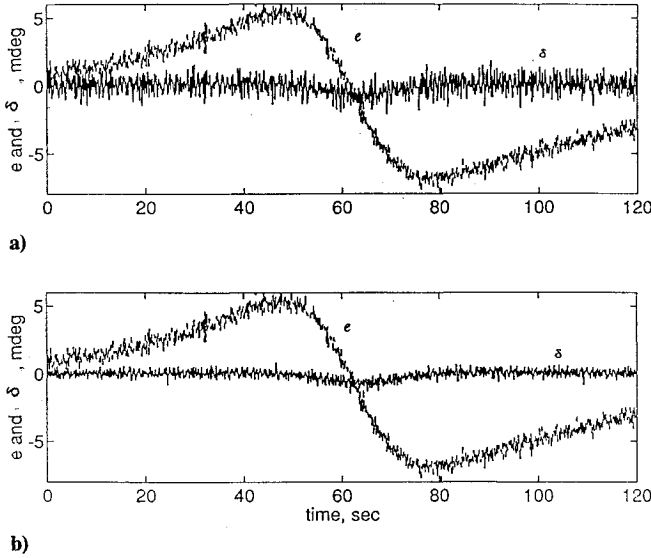


Fig. 18 Elevation tracking error (solid line) and noisy monopulse signal (dashed line): a) without averaging and b) with averaging.

of the nonaveraged process (σ_N^2) by the factor N ,

$$m_{av} = m_N, \quad \sigma_{av}^2 = \sigma_N^2 / N \quad (17)$$

Define r_s , the ratio of variances of nonaveraged and averaged signal, as

$$r_s = \sigma_N^2 / \sigma_{av}^2 \quad (18a)$$

and its logarithmic counterpart, an SNR increase, ΔSNR , as

$$\Delta SNR = SNR_{av} - SNR = 10 \log_{10} r_s \quad [\text{dB}] \quad (18b)$$

Then for white noise, from definition (12), one obtains $r_s = N$ and $\Delta SNR = 10 \log_{10} N$ decibels.

Consider high-frequency noise with a constant spectrum within the interval $[f_0, f_c]$, such that $0 < f_0 < f_c$, f_0 is a cut-off frequency (the lowest frequency component of the noise) and f_c is the Nyquist frequency $f_c = 0.5/\Delta t$. Results of noise reduction for the high-frequency noise, obtained through simulations, are shown in Fig. 17. From this ratio r_s vs the cut-off frequency f_0 plot it is evident that the high-frequency noise is more suppressed through averaging than the white noise (r_s increases from 5 in the case of white noise to 50 in the case of high-frequency noise for cut-off frequencies of 8 Hz and higher, and SNR increase, ΔSNR , is from 7 to 17 dB, respectively). These results have also been confirmed by simulations of monopulse tracking with $SNR = 20$ dB, where the elevation pointing error for the case of nonaveraged signal is shown in Fig. 18a, and the same error for the averaged signal is shown in Fig. 18b, with the noise power ratio $r_s = 4.7$ (SNR increase $\Delta SNR = 6.7$ dB), which is close to the predicted $r_s = 5$ ($\Delta SNR = 7$ dB).

Conclusions

It has been shown that the feedforward upgrade of the existing DSN antenna servos improves tracking at higher rates and that monopulse tracking is an appropriate replacement of the conscan technique for the considered rates. A sampling rate of 10 Hz is satisfactory to maintain the accuracy of the monopulse control system, and the 0.1-s delay does not deteriorate the system performance. Repositioning of the elevation pinion or implementation of a notch filter or adjustment of amplifier gain serves as a tool for improving tracking accuracy. The monopulse SNR is improved through averaging the high-frequency sampled signal.

Appendix: Proof of Eq. (17)

The first part of Eq. (17) follows from the definition of the averaged process (16) and the equality of mean values in the cluster, Eq. (15), namely,

$$m_{av} = \frac{1}{N} \sum_{k=1}^N E[u_k(i\Delta T)] = \frac{1}{N} \sum_{k=1}^N m_k = m_N \quad (A1)$$

In order to prove the second part of Eq. (17), denote

$$\Delta u_k(i\Delta T) = u_k(i\Delta T) - u_0(N\Delta t) = n_k(i\Delta T) \quad (A2)$$

$$n_k(i\Delta T) = n(i\Delta T + k\Delta t), \quad k = 1, \dots, N \quad (A3)$$

$$\Delta u_{av}(i\Delta T) = \frac{1}{N} \sum_{k=1}^N \Delta u_k(i\Delta T) \quad (A4)$$

Thus the variance of the averaged process is

$$\sigma_{av}^2 = E[\Delta u_{av}(i\Delta T)]^2 = \frac{1}{N^2} \sum_{k,l=1}^N E[\Delta u_k(i\Delta T) \Delta u_l(i\Delta T)] \quad (A5)$$

The white noise is not correlated, that is,

$$E[n_k(i\Delta T) n_l(i\Delta T)] = 0 \quad \text{for} \quad k \neq l \quad (A6)$$

Therefore

$$E[\Delta u_k(i\Delta T) \Delta u_l(i\Delta T)] = 0 \quad \text{for} \quad k \neq l \quad (A7)$$

Introducing Eq. (A7) and Eq. (A15) into Eq. (A5),

$$\sigma_{av}^2 = \frac{1}{N^2} \sum_{k=1}^N E[\Delta u_k(i\Delta T)]^2 = \frac{1}{N^2} \sum_{k=1}^N \sigma_k^2 = \frac{\sigma_N^2}{N} \quad (A8)$$

proves the second part of Eq. (17).

Acknowledgments

This research was performed at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. The authors would like to thank Ben Parvin for his support and encouragement.

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